On Timed Automata with Discrete Time—Structural and Language Theoretical Characterization

Herrman Gruber and Markus Holzer and Astrid Kiehn
Institut für Informatik, Technische Universität München,
Boltzmannstraße 3, D-85748 Garching bei München, Germany
e-mail: {gruberh,holzer,kiehn}@in.tum.de

and

Barbara König
Institut für Formale Methoden der Informatik, Universität Stuttgart,
Universitätsstraße 38, D-70569 Stuttgart, Germany
e-mail: koenigb@fmi.uni-stuttgart.de

ABSTRACT

Timed automata were introduced in [1] in order to model real-time systems from a quantitative perspective, to make specification and verification of models of real-time systems easier. Simply speaking, a timed automaton is a finite ω-automata with a finite set of clocks, which can be independently reset, and where the automaton can keep track of the time elapsed since the last reset. Since then, many generalizations of timed automata were investigated as, e.g., timed automata with silent transitions [3], or timed automata with periodic clock constraints [4]. It was shown that these extensions are equally powerful and strictly increase the expressive power of timed automata, regardless on the used time semantics, i.e., the interpretation of the clocks, which assigns some value to each clock. There are two major semantics discussed in the literature, namely, the discrete time and the dense-time model. Although, the latter is more natural, from the physical point of view, and allows one an easy modelling of real-time systems, it requires decision methods for real time logics, which in general are undecidable [2]. In fact, the class of timed languages accepted by automata interpreted under the dense-time model is closed under all positive Boolean operations, but is not closed under complementation [1]. The latter is also true for timed languages over discrete time, except for the complementation closure, which was shown in [5] in terms of a variant of monadic second order logic.

We give a structural characterization of the classes of timed languages with or without periodic clock constraints (or equivalently with our without silent transitions). To this end we transform the timed automaton into a so-called tick automaton. Intuitively, a tick automaton is a Büchi automaton where the input alphabet is equipped with an additional "clock-tick"-symbol ($\surd$) modelling the discrete flow of time. Compared to the discrete time semantics introduce in [1], the tick automaton approach is more general, since it allows (finite or infinite) stuttering of actions on a particular time stamp during the computation. It turns out that any tick automaton induced by a timed automaton has a particular $\surd$-transition structure, namely deterministic tick-paths ending in (trivial or non-trivial) tick-loops. On the other hand, we can show that any tick automaton (not necessarily induced by a timed automaton) can be converted

into this normal form. This nicely corresponds to the intuitive behaviour of time as a sort of deterministic or non-branching flow. It is worth mentioning, that timed automaton in general induce non-trivial tick-loops, while the tick-loops become trivial for aperiodic timed automata. Moreover, besides this structural characterization of timed languages in terms of tick automata, we provide a language theoretical characterization by a simple condition, which will be called $\surd$-stretchy. This condition is closely related to the aperiodic condition of regular languages on finite words. Loosely speaking, $\surd$-stretchyness tells us that tick actions in direct sequence can only be counted up to a given threshold, which depends on the language only. We hope that these characterizations give some further insights into the behaviour of timed languages. Moreover, the provided characterizations can be used to show an alternative way to reprove the complementation closure of timed aperiodic languages, now in the framework of ω-automata, thus reproving the result given in [5]. All presented results are effectively constructible, and therefore can be used for the verification of discrete timed systems.

References

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